Section 1.5: Linear Models

An **asset** is an item owned that has value.
Linear Depreciation refers to the amount of decrease in the book value of an asset.
The **purchase price**, also known as **original cost**, of an asset is the price paid for the asset when purchased.
The **scrap value** of an asset is the remaining value after it is no longer seen as useable.

Example 1: In 2010, the B&C Company installed a new machine in one of its factories at a cost of $150,000. The machine is depreciated linearly over 15 years with no scrap value.

a. Find the rate of depreciation for this machine.

b. Find an expression for the machine's book value in the t-th year of use ($0 \le t \le 15$).

Example 2: A company's car has an original value of $85, 000 and will be depreciated linearly over 6 years with scrap value of $10,000.

a. Find the expression giving the book value of the car at the end of year t ($0 \le t \le 6$).

b. Find the car's book value in at the end of the third year.

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Linear Cost, Revenue and Profit Functions:

If x is the number of units of a product manufactured or sold at a firm then,

The **cost function**, C(x), is the total cost of manufacturing x units of the product.

Fixed costs are the costs that remain regardless of the company's activity.

   Examples: building fees (rent or mortgage), executive salaries

Variable costs are costs that vary with the production or sales.

   Examples; wages of production staff, raw materials

The **revenue function**, R(x), is the total revenue realized from the sale of x units of the product.

The **profit function**, P(x), is the total profit realized from the manufacturing and sale of the x units of product.

Formulas: Suppose a firm has fixed cost of F dollars, production cost of c dollars per unit and selling price of s dollars per unit then

C(x) =

R(x) =

P(x) =

Where x is the number of units of the commodity produced and sold.

Example 3: A manufacturer has a monthly fixed cost of $150,000 and a production cost of $18 for each unit produced. The product sells for $24 per unit.

a. What is the cost function?

b. What is the revenue function?

c. What is the profit function?

d. Compute the profit (loss) corresponding to production levels of 22,000 and 28,000.

e. How many units must the company produce and sell if they wish to make a profit of $40,000?

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Example 4: Auto Time, a manufacturer of 24-hour variable timers, has a fixed monthly cost of $56000 and a production cost of $10 per unit manufactured. The timers sell for $17 each.

a. What is the cost function?

b. What is the revenue function?

c. What is the profit function?

d. Compute the profit (loss) corresponding to the production and sale of 4,000, 8,000 and 10,000 timers.

Break-Even Point

The **break-even level of operation-** is when the company neither makes a profit nor sustains a loss.
Note: The break-even level of operation is represented by the point of intersection of two lines.
The break-even level of production means the profit is zero.

Consider the following graph.

The point (x_o , y_o) is referred to as the break-even point.
x_o = break even quantity
y_o = break even revenue

If $x < x_o$ then $R(x) < C(x)$, therefore $P(x) = R(x) - C(x) < 0$ so you will have a loss.
If $x > x_o$ then $R(x) > C(x)$, therefore $P(x) = R(x) - C(x) > 0$ so you will have a profit.

Example 5: Find the break-even quantity and break-even revenue if $C(x) = 110x + 40,000$ and $R(x) = 150x$.

Example 6: The XYZ Company has a fixed cost of 20,000, a production cost of $12 for each unit produced and a selling price of $20 for each unit produced.

a. Find the break-even point for the firm.

b. If the company produces and sells 2000 units, would they obtain a profit or loss?

c. If the company produces and sells 3000 units, would they obtain a profit or loss?

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Example 7: Given the following profit function P(x) = 6x -12,000.

a. How many units should be produced in order to realize a profit of $9,000?

b. What is the profit or loss if 1,000 units are produced?

Example 8: A bicycle manufacturer experiences fixed monthly costs of $124,992 and variable costs of $52 per standard model bicycle produced. The bicycles sell for $100 each. How many bicycles must he produce and sell each month to break even? What is his total revenue at the point where he breaks even?